

Math 53: Worksheet 4

October 16

1. Find all the relative maxima and minima, if any, of the following functions:
 - (a) $f(x, y) = x^3 - 2xy + 4y$.
 - (b) $f(x, y) = ye^x - 3x - y + 5$.
2. Use Lagrange multipliers to find the maximum and/or minimum values of the function subject to the given constraints:
 - (a) $f(x, y, z) = xyz$ subject to $x^2 + 2y^2 + 3z^2 = 6$.
 - (b) $f(x, y, z, t) = x + y + z + t$ subject to $x^2 + y^2 + z^2 + t^2 = 1$.
3. A wire 10 inches long is to be cut into two pieces. A circle is to be formed from one piece and a square from the other. How should the wire be cut to **minimize** the total area enclosed by the two figures? (Use Lagrange multipliers even though this problem can be done using single-variable calculus)
4. You are part of a team that has discovered a new spherical planet of radius 6 metres. Based on a coordinate system which has its origin at the centre of the planet, the temperature is given by $T(x, y, z) = 6y - x^2 + yz + 40$. Find the coldest point on the surface of this planet.
5. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is 72 cm.
6. The company Victorinox manufactures Swiss Army knives. The number of knives it can produce in a day with L units of labour and K units of capital is given by the production function $P(L, K) = 500L^{3/5}K^{2/5}$. The expenses are \$100 per day for each unit of labour and \$200 per day for each unit of capital. Total expenses are limited to \$1000 per day. How many units of labour and capital should be utilized to maximize production?
7. The base of an aquarium of volume 2m^3 is made of slate and the sides are made of glass. If slate costs five times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials.